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LETTER TO THE EDITOR

A determinant representation for a correlation function of the scaling Lee–Yang model

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Abstract. We consider the scaling Lee–Yang model. It corresponds to the unique perturbation of the minimal CFT model $M_{2,5}$. This model is not unitary. We are using an expression for form factors in terms of symmetric polynomials in order to obtain a closed expression for the correlation function of the trace of the energy-momentum tensor. This expression is a determinant of an integral operator. Similar determinant representations were proven to be useful not only for quantum correlation functions but also in matrix models.

1. Introduction

The theory of massive, relativistic, integrable models is an important part of modern quantum field theory [21–28]. Scattering matrices in these models factorize into a product of two-body S-matrices [21]. Form factors can be calculated on the basis of a bootstrap approach [21–28].

The purpose of this letter is to calculate correlation functions. As usual, correlation functions can be represented as an infinite series of form factor contributions. In this letter we sum up all these contributions and obtain a closed expression for the correlation function of the energy-momentum tensor. We follow the approach of [5]. We introduce an auxiliary Fock space and auxiliary Bose fields (we shall call them dual fields). These fields help us to represent the form factor decomposition of the correlation function in a form similar to the 'free fermionic' case. This approach was developed in [29, 30, 10]. Finally, the correlation function is represented as a vacuum mean value (in the auxiliary Fock space) of a determinant of an integral operator (4.3). The determinant representation is the basis for a nonperturbative analysis of correlation functions. It provides the opportunity to describe correlation functions by differential equations, to calculate asymptotics and to discover hidden symmetries of the model. Painlevé differential equations were obtained for correlation functions of the Ising model [6] and of the impenetrable Bose gas [7] on the basis of the determinant representation. Later it was shown that a similar Painlevé equation describes correlation in XXZ Heisenberg spin chain at magnetic field close to critical [12]. In sl_2 Gaudin model a generating function of correlators was represented as a determinant [15]. The determinant representation is the unique method of calculation of time- and temperature-dependent correlation functions (for moderate temperatures). This

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method allowed us to calculate time- and temperature-dependent correlation fucntions in the Bose gas with delta interaction [13], in the one-dimensional (1D) quantum Ising model [16], in the XY model [11] and in the Hubbard model [8]. Determinant representations were proven to be useful not only for the theory of quantum correlation functions [6–16] but also in matrix models [17–20]. In matrix models the determinant representation provides a natural method for the description of level spacing.

The scaling Lee–Yang model can be described by the $\phi_{1,3}$-perturbation of the non-unitary minimal model $M_{2,5}$ [1]. The theory is known to be integrable and is described by factorizable scattering theory of only one kind of particle of mass m. The two-body scattering amplitude is given by [1]

$$S(\beta) = \frac{\sinh\beta + \mathrm{i}\sin(2\pi/3)}{\sinh\beta - \mathrm{i}\sin(2\pi/3)}. \tag{1.1}$$

The pole at $\beta = 2\pi\mathrm{i}/3$ corresponds to a bound state. The residue at the pole is negative. This means that, as a consequences of non-unitarity the three-point coupling is imaginary.

The S-matrix can be obtained from the breather–breather S-matrix of the sine Gordon model [2]. The simplest example of breather–breather S-matrix of the sine Gordon model was first calculated in [3].

Form factors for the trace of the energy-momentum tensor $\Theta = T_\mu^\mu/4$ are defined as matrix elements between the vacuum state $\langle\mathrm{vac}\,|$ and n particle states characterized by rapidities β_i ($i = 1, \ldots, n$):

$$F_n(\beta_1, \ldots, \beta_n) = \langle\mathrm{vac}\,|\Theta(0)|\beta_1, \ldots, \beta_n\rangle. \tag{1.2}$$

The multiparticle form factors F_n were calculated in [2, 4]:

$$F_n(\beta_1, \ldots, \beta_n) = H_n Q_n(x_1, \ldots, x_n) \prod_{i<j} \frac{f(\beta_i - \beta_j)}{x_i + x_j} \tag{1.3}$$

where $x_i = \mathrm{e}^{\beta_i}$, $i = 1, \ldots, n$,

$$H_n = -\frac{\pi m^2}{4\sqrt{3}} \left(\frac{\mathrm{i}3^{1/4}}{2^{1/2}v(0)}\right)^n. \tag{1.4}$$

The function $f(\beta)$ is given by

$$f(\beta) = \frac{\cosh\beta - 1}{\cosh\beta + \frac{1}{2}} v(\mathrm{i}\pi - \beta) v(-\mathrm{i}\pi + \beta) \tag{1.5}$$

where

$$v(\beta) = \exp\left(2\int_0^\infty \frac{\mathrm{d}t}{t} \mathrm{e}^{\mathrm{i}\beta t/\pi} \frac{\sinh(t/2)\sinh(t/3)\sinh(t/6)}{\sinh^2 t}\right). \tag{1.6}$$

The function $f(\beta)$ has a single pole in the strip $0 \leqslant \mathrm{Im}\beta < \pi$ at $\beta = 2\pi\mathrm{i}/3$ and a single zero at $\beta = 0$.

We shall use the elementary symmetric polynomials $\sigma_k(x_1, \ldots, x_n)$, which are defined as

$$\prod_{j=1}^n (x + x_j) = \sum_{k\in\mathbb{Z}} x^{n-k} \sigma_k(x_1, \ldots, x_n).$$

It is important to bear in mind the fact that σ_k is equal to zero if $k < 0$ or $k > n$. The symmetric polynomials $Q_n(x_1, \ldots, x_n)$ arising in the formula for form factors can be written as:

$$Q_0 = 1$$
$$Q_1 = 1$$
$$Q_2 = \sigma_1 \tag{1.7}$$
$$Q_n = \sigma_1 \sigma_{n-1} P_n \qquad n \geqslant 3$$

where

$$P_n = \det_{n-3}(\Sigma_{ij}). \tag{1.8}$$

Here Σ_{ij} is a matrix of dimension $(n-3)$. Its entries are equal to

$$\Sigma_{ij} = \sigma_{3i-2j+1} \qquad 1 \leqslant i, j \leqslant n-3. \tag{1.9}$$

The index $n-3$ in the expression $\det_{n-3}$ denotes the dimension of the matrix Σ_{ij}.

After Wick rotation to the Euclidean space, the correlation function of the operator Θ can be presented as an infinite series of form factors contributions

$$\langle \Theta(x)\Theta(0)\rangle = \sum_{n=0}^{\infty} \int \frac{\mathrm{d}^n\beta}{n!(2\pi)^n} \langle \mathrm{vac}\,|\Theta(0)|\beta_1, \ldots, \beta_n\rangle\langle\beta_n, \ldots, \beta_1|\Theta(0)|\,\mathrm{vac}\rangle \times \exp\left[-mr\sum_{j=1}^{n}\cosh\beta_j\right] \tag{1.10}$$

where $r = (x^\mu x_\mu)^{1/2}$.

In this letter we sum up this series explicitly. This letter is organized as follows. Section 2 is devoted to a transformation of the form factors to a form, which is convenient for summation. In section 3 we introduce auxiliary quantum operators—dual fields—in order to factorize an expression for the correlation function and to represent it in a form similar to the 'free fermionic' case. In section 4 we sum up the series (1.10) to a Fredholm determinant. In section 5 we consider an example, which illustrates how to use the Fredholm determinant representation and dual fields.

2. A transformation of the form factor

A determinant of a linear integral operator $I + V$ can be decomposed into a Taylor series:

$$\det(I+V) = \sum_{n=0}^{\infty} \int \frac{\mathrm{d}x_1 \cdots \mathrm{d}x_n}{n!} \det_n \begin{pmatrix} V(x_1,x_1) & \cdots & V(x_1,x_n) \\ V(x_2,x_1) & \cdots & V(x_2,x_n) \\ \cdot & \cdot & \cdot \\ V(x_n,x_1) & \cdots & V(x_n,x_n) \end{pmatrix}. \tag{2.1}$$

In order to obtain a determinant representation for the correlation function we shall represent the form factor expansion (1.10) in the form (2.1). Determinants of integral operators, which we consider can be called Fredholm determinants.

The form factors (1.3) are proportional to the polynomials Q_n (1.7). The polynomial Q_n for $n \geqslant 3$ is proportional to the determinant of the matrix Σ_{ij} (1.9).

Let us define a new matrix M:

$$M_{ij} = \sigma_{3i-2j-1} \qquad 1 \leqslant i, j \leqslant n. \tag{2.2}$$

The dimension of the matrix is n. Note that $M_{1j} = \delta_{1,j}$, $M_{2j} = \delta_{1,j}\sigma_3 + \delta_{2,j}\sigma_1$ and $M_{nj} = \delta_{n,j}\sigma_{n-1}$ for $j = 1, \ldots, n$. Using the relation $M_{ij} = \Sigma_{i-2,j-2}$ $(i, j = 3, \ldots, n-1)$, one can show that

$$Q_n = \det_n M \qquad n \geqslant 0. \tag{2.3}$$

The matrix M_{ij} contains n^2 different functions, depending on the same set of arguments $x_1, \ldots, x_n$.

The main aim of this and the next sections is to transform the matrix (2.2) to such a form, that entries of a new matrix would be parametrized by a single function, depending on different sets of variables (like $V(x_i, x_j)$ in (2.1))

$$M_{ij} \to \hat{D}_{ij} \qquad \hat{D}_{ij} = \hat{D}(x_i, x_j). \tag{2.4}$$

In order to study correlation functions, we need to find the square of the polynomials Q_n:

$$Q_n^2 = \det_n(C_{ij}) \tag{2.5}$$

where

$$C_{jk} = (M^T M)_{jk} = \sum_{i=1}^{n} \sigma_{3i-2j-1}\sigma_{3i-2k-1} = \sum_{i=-\infty}^{n} \sigma_{3i-2j-1}\sigma_{3i-2k-1}. \tag{2.6}$$

Here we used the relation $\sigma_k = 0$ for $k < 0$.

Note that the elementary symmetric polynomials can be expressed as:

$$\sigma_k = \frac{1}{2\pi \mathrm{i}} \oint_\gamma \frac{\mathrm{d}z}{z^{n-k+1}} \prod_{m=1}^{n} (z + x_m) \tag{2.7}$$

where the integration contour γ is a circle around the origin in the positive direction.

Substituting the above expression into (2.6) and summing up the infinite series, we have

$$C_{jk} = \oint_\gamma \frac{\mathrm{d}z_1}{2\pi \mathrm{i}} \oint_\gamma \frac{\mathrm{d}z_2}{2\pi \mathrm{i}} \frac{z_1^{2n-2j+1} z_2^{2n-2k+1}}{(z_1 z_2)^3 - 1} \prod_{m=1}^{n} (z_1 + x_m)(z_2 + x_m). \tag{2.8}$$

Here we have chosen the radius of the circle γ to be greater than one in order for the series to converge.

The matrix C still depends on n^2 different functions C_{jk}. However, this matrix can be transformed to a more convenient form. Let us introduce the following matrix

$$A_{jk} = \frac{1}{(n-j)!} \frac{\mathrm{d}^{n-j}}{\mathrm{d}(x^2)^{n-j}} \prod_{m \neq k}^{n} (x^2 + x_m^2) \bigg|_{x^2=0} \tag{2.9}$$

which has a determinant

$$\det A = \prod_{i<j}^{n} (x_i^2 - x_j^2). \tag{2.10}$$

Using this matrix, we define another matrix D, which differs from C by a linear transformation

$$D = A^T C A. \tag{2.11}$$

We have an explicit expression for matrix elements of D:

$$D_{jk} = \frac{1}{(2\pi \mathrm{i})^2} \oint \mathrm{d}^2 z \frac{z_1 z_2}{z_1^3 z_2^3 - 1} Y(z_1, x_j) Y(z_2, x_k) \tag{2.12}$$

where

$$Y(z, x) = \frac{J(z)}{z^2 + x^2} \tag{2.13}$$

and

$$J(z) = \prod_{a=1}^{n}(z + x_a)(z^2 + x_a^2). \tag{2.14}$$

Taking the integral with respect to z_2, we have (after the symmetrization of the integrand)

$$D_{jk} = \frac{1}{18\pi \mathrm{i}} \oint_\gamma \frac{\mathrm{d}z}{z} \left(\sum_{l=1}^{3} \omega^l Y(\omega^{-l} z, x_j) \right) \left(\sum_{m=1}^{3} \omega^m Y(\omega^{-m} z^{-1}, x_k) \right). \tag{2.15}$$

Here $\omega = \mathrm{e}^{2\pi \mathrm{i}/3}$ and the integration contour γ is a circle whose radius is larger than 1. Explicitly, the sum over l can be written as

$$\sum_{l=1}^{3} \omega^l Y(\omega^{-l} z, x) = Y(z, x) + \omega Y(\omega^{-1} z, x) + \omega^{-1} Y(\omega z, x). \tag{2.16}$$

The determinants of matrices C and D are related by

$$\det{}_n C = \prod_{i<j}^{n} (x_i^2 - x_j^2)^{-2} \det{}_n D. \tag{2.17}$$

Thus, we obtain a determinant representation for Q_n^2,

$$Q_n^2 = \frac{\det_n D}{\prod_{i<j}^{n} (x_i^2 - x_j^2)^2}. \tag{2.18}$$

3. Dual fields

The entries of the matrix D_{jk} are parametrized now by a single function D. However, the element D_{jk}, is not yet a function of only two arguments, because of the product $J(z) = \prod_{m=1}^{n}(z + x_m)(z^2 + x_m^2)$. This product depends on all x_m. In order to omit this product we introduce an auxiliary Fock space and auxiliary quantum operators—dual fields.

Let us define

$$\Phi_1(x) = q_1(x) + p_2(x) \qquad \Phi_2(x) = q_2(x) + p_1(x) \tag{3.1}$$

where the operators $p_j(x)$ and $q_j(x)$ act on the canonical Bose Fock space in the following way

$$(0|q_j(x) = 0 \qquad p_j(x)|0) = 0. \tag{3.2}$$

Non-zero commutators are given by

$$[p_1(x), q_1(y)] = [p_2(x), q_2(y)] = \xi(x, y) = \log((x + y)(x^2 + y^2)). \tag{3.3}$$

Due to the symmetry of the function $\xi(x, y) = \xi(y, x)$, all fields $\Phi_j(x)$ commute with each other

$$[\Phi_j(x), \Phi_k(y)] = 0 \qquad j, k = 1, 2. \tag{3.4}$$

Dual fields are linear combinations of canonical Bose fields, see [10, p 210].

Instead of $Y(z, x)$ let us define an operator-valued function

$$\hat{Y}(z, x) = \frac{\mathrm{e}^{\Phi_1(z)}}{z^2 + x^2}. \tag{3.5}$$

Instead of D_{jk} (2.15), we shall introduce an operator in the auxiliary Fock space,

$$\hat{D}(x, y) = \frac{1}{18\pi \mathrm{i}} \oint \frac{\mathrm{d}z}{z} \left(\sum_{l=1}^{3} \omega^l \hat{Y}(\omega^{-l} z, x) \right) \left(\sum_{m=1}^{3} \omega^m \hat{Y}(\omega^{-m} z^{-1}, y) \right). \tag{3.6}$$

It is easy to show that an exponent of dual field acts like a shift operator. Namely, if $g(\Phi_1(y))$ is a function of $\Phi_1(y)$ then

$$\left(0\middle|\prod_{m=1}^{n} \mathrm{e}^{\Phi_2(x_m)} g(\Phi_1(y))\middle|0\right) = \left(0\middle|g\left(q_1(y) + \sum_{m=1}^{n} \xi(x_m, y)\right)\middle|0\right) = g(\log J(y)).$$

Using this property of dual fields one can remove the products $J(z)$ from the matrix D_{jk}. For a more detailed derivation one should refer to formula (3.6) of [5].

Standard arguments of quantum field theory show that

$$\det{}_n D = (0|\det{}_n(\hat{D}(x_j, x_k)\mathrm{e}^{\frac{1}{2}\Phi_2(x_j)+\frac{1}{2}\Phi_2(x_k)})|0). \tag{3.7}$$

Heretofore, we have written the Q_n^2 factor of $|F_n|^2$ as a determinant. The absolute value of the form factor is equal to

$$\begin{aligned} |F_n(\beta_1, \ldots, \beta_n)|^2 &= |H_n|^2 Q_n^2 \prod_{i<j} \left|\frac{f(\beta_i - \beta_j)}{(x_i + x_j)}\right|^2 \\ &= |H_n|^2 \det{}_n D \prod_{i<j} \left|\frac{f(\beta_i - \beta_j)}{(x_i^2 - x_j^2)(x_i + x_j)}\right|^2. \end{aligned} \tag{3.8}$$

In order to factorize the double product part, we introduce another dual field

$$\tilde{\Phi}_0(x) = \tilde{q}_0(x) + \tilde{p}_0(x). \tag{3.9}$$

As usual

$$(0|\tilde{q}_0(x) = 0 \qquad \tilde{p}_0(x)|0) = 0. \tag{3.10}$$

The operators $\tilde{q}_0(x)$ and $\tilde{p}_0(y)$ commute with all p_j and q_j ($j = 1, 2$). The only non-zero commutator is

$$[\tilde{p}_0(x), \tilde{q}_0(y)] = \eta(x, y) \tag{3.11}$$

where

$$\eta(x, y) = \eta(y, x) = 2\log\left|\frac{f(\log\frac{x}{y})}{(x^2 - y^2)(x + y)}\right|. \tag{3.12}$$

Here we have used the fact that $|f(\beta)|$ is a symmetric function, $|f(-\beta)| = |f(\beta)|$. It is worth mentioning that the right-hand side of (3.12) has no singularity at $x = y$, because $f(\beta)$ has a zero of first order at $\beta = 0$. Using some of the equations for minimal form factor from [4]

$$f(\beta)f(\beta + \mathrm{i}\pi) = \frac{\sinh\beta}{\sinh\beta - \mathrm{i}\sin(\pi/3)} \tag{3.13}$$

and $f(\mathrm{i}\pi) = 4v^2(0)$, we can see that

$$f'(0) = \frac{\mathrm{i}}{2\sqrt{3}v^2(0)}. \tag{3.14}$$

Hence

$$\eta(x, x) = -2\log|\lambda x^3| \tag{3.15}$$

where

$$\lambda = 8\sqrt{3}v^2(0). \tag{3.16}$$

The newly introduced dual field also mutually commute

$$[\tilde{\Phi}_0(x), \tilde{\Phi}_0(y)] = 0 = [\tilde{\Phi}_0(x), \Phi_j(y)]. \tag{3.17}$$

Due to the Campbell–Hausdorff formula, we have

$$\left(0\left|\prod_{m=1}^{n} e^{\tilde{\Phi}_0(x_m)}\right|0\right) = \prod_{i,j=1}^{n} e^{\frac{1}{2}\eta(x_i,x_j)} = \lambda^{-n}\prod_{m=1}^{n} x_m^{-3}\prod_{i<j}^{n}\left|\frac{f(\log\frac{x_i}{x_j})}{(x_i^2-x_j^2)(x_i+x_j)}\right|^2. \tag{3.18}$$

Combining these results, we can represent the square of the absolute value of the form factor as a determinant:

$$|F_n(\beta_1,\ldots,\beta_n)|^2 = \left(\frac{\pi m^2}{4\sqrt{3}}\right)^2 12^n (0|\det_n(x_j^{3/2}x_k^{3/2}\hat{D}(x_j,x_k)e^{\frac{1}{2}\Phi_0(x_j)+\frac{1}{2}\Phi_0(x_k)})|0).$$

Here

$$\Phi_0(x) = \tilde{\Phi}_0(x) + \Phi_2(x). \tag{3.19}$$

So we managed to represent the square of the absolute value of the form factor as a determinant, similar to one of the terms on the right-hand side of (2.1). In the next section we shall sum up all contributions of the form factors and obtain a determinant representation for the correlation function.

4. The determinant representation

Because the scaling Lee–Yang model is a non-unitary theory, the normalization constant H_n (1.4) is pure imaginary for n odd. This leads to the following relation:

$$\langle \mathrm{vac}|\Theta(0)|\beta_1,\ldots,\beta_n\rangle\langle\beta_n,\ldots,\beta_1|\Theta(0)|\mathrm{vac}\rangle = (-1)^n|F_n(\beta_1,\ldots,\beta_n)|^2. \tag{4.1}$$

Then the correlation function of Θ can be written as

$$\begin{aligned}\langle\Theta(x)\Theta(0)\rangle &= \sum_{n=0}^{\infty}(-1)^n\int\frac{d^n\beta}{n!(2\pi)^n}|F_n(\beta_1,\ldots,\beta_n)|^2\prod_{j=1}^{n}e^{-\theta(x_j)}\\ &= \left(\frac{\pi m^2}{4\sqrt{3}}\right)^2\left(0\left|\sum_{n=0}^{\infty}\int_0^{\infty}\frac{d^n x}{n!}\left(-\frac{6}{\pi}\right)^n\right.\right.\\ &\quad\left.\left.\times\det_n(x_jx_k\hat{D}(x_j,x_k)e^{\frac{1}{2}(\Phi_0(x_j)+\Phi_0(x_k))}e^{-\frac{1}{2}(\theta(x_j)+\theta(x_k))})\right|0\right).\end{aligned}$$

where

$$\theta(x) = \frac{mr}{2}(x+x^{-1}). \tag{4.2}$$

Now we can use (2.1) in order to sum up. Finally, we obtain a determinant representation in terms of an integral operator (Fredholm determinant)

$$\langle\Theta(x)\Theta(0)\rangle = \left(\frac{\pi m^2}{4\sqrt{3}}\right)^2(0|\det(I-\hat{U})|0). \tag{4.3}$$

The operator $\hat{U}$ acts on a function $g(x)$ as

$$[\hat{U}g](x) = \int_0^{\infty}dy\,\hat{U}(x,y)g(y). \tag{4.4}$$

The kernel of the integral operator $\hat{U}(x, y)$ is equal to

$$\hat{U}(x, y) = \frac{6}{\pi} xy\hat{D}(x, y)\mathrm{e}^{\frac{1}{2}(\Phi_0(x)+\Phi_0(y))}\mathrm{e}^{-\frac{1}{2}(\theta(x)+\theta(y))} \tag{4.5}$$

where $\hat{D}$ is given by (3.6)

$$\hat{D}(x, y) = \frac{1}{18\pi\mathrm{i}} \oint \frac{\mathrm{d}z}{z} \left(\sum_{l=1}^{3} \omega^l \hat{Y}(\omega^{-l} z, x) \right) \left(\sum_{m=1}^{3} \omega^m \hat{Y}(\omega^{-m} z^{-1}, y) \right) \tag{4.6}$$

and

$$\hat{Y}(z, x) = \frac{\mathrm{e}^{\Phi_1(z)}}{z^2 + x^2}. \tag{4.7}$$

The dual fields $\Phi_0(x)$ and $\Phi_1(x)$ were defined in the section 3 (see (3.1) and (3.9)). The main property of these dual fields is that they commute with each other, therefore the Fredholm determinant $\det(I - \hat{U})$ is well defined. The $\det(I - \hat{U})$ is an operator in auxiliary Fock space and it also belongs to an Abelian subalgebra. On the other hand, the vacuum expectation value of these operators is non-trivial. It follows from the commutation relations (3.3), (3.11), that in order to calculate the vacuum expectation value, one should use the following prescription

$$\left(0 \left| \prod_{a=1}^{M_1} \mathrm{e}^{\Phi_0(x_a)} \prod_{b=1}^{M_2} \mathrm{e}^{\Phi_1(x_b)} \right| 0 \right) = \prod_{a=1}^{M_1} \prod_{b=1}^{M_1} \mathrm{e}^{\frac{1}{2}\eta(x_a, x_b)} \prod_{a=1}^{M_1} \prod_{b=1}^{M_2} \mathrm{e}^{\xi(x_a, x_b)}. \tag{4.8}$$

Here

$$\eta(x, y) = 2\log \left| \frac{f(\log \frac{x}{y})}{(x^2 - y^2)(x + y)} \right| \tag{4.9}$$

and

$$\xi(x, y) = \log((x + y)(x^2 + y^2)). \tag{4.10}$$

We saw that an introduction of an auxiliary Fock space helped us to calculate the correlation function. It would be interesting to understand a relation of the auxiliary Bose fields to vertex operators which appear in an alternative expression for correlation functions [31] and form factors [26] based on quantum group approach.

5. Large r-asymptotic

In order to illustrate how to handle dual quantum fields we shall rederive the long-distance asymptotic from the determinant representation.

Using the technique developed in [5], we calculate the long-distance asymptotic of the correlation function.

The determinant can be written as

$$\det(I - \hat{U}(x, y)) = \det(I - \tilde{U}(z_1, z_2)) \tag{5.1}$$

where

$$\tilde{U}(z_1, z_2) = \int_0^\infty \mathrm{d}x\, P_1(z_1, x) P_2(z_2, x) \tag{5.2}$$

and

$$P_1(z,x) = \frac{x}{3\pi^2 \mathrm{i} z}\left(\sum_{l=1}^{3}\omega^l \hat{Y}(\omega^{-l}z, x)\right)\mathrm{e}^{\frac{1}{2}\Phi_0(x)-\frac{1}{2}\theta(x)} \tag{5.3}$$

$$P_2(z,y) = y\left(\sum_{l=1}^{3}\omega^l \hat{Y}(\omega^{-l}z^{-1}, y)\right)\mathrm{e}^{\frac{1}{2}\Phi_0(y)-\frac{1}{2}\theta(y)}. \tag{5.4}$$

The integral operator $\tilde{U}(z_1, z_2)$ acts on a function $g(z)$ as

$$[\tilde{U}g](z_1) = \oint \tilde{U}(z_1, z_2) g(z_2)\mathrm{d}z_2. \tag{5.5}$$

Here the integration contour is a circle around zero in positive direction.

In the limit $r \to \infty$, we evaluate the integral (5.2) by the saddle-point method. The saddle point of the function $\theta(x)$ is $x = 1$. Hence, we can estimate the integral in (5.2) as

$$\tilde{U}(z_1, z_2) = P_1(z_1, 1)P_2(z_2, 1)\left(\sqrt{\frac{2\pi}{mr}} + \mathrm{O}(r^{-3/2})\right). \tag{5.6}$$

Thus, for the large r asymptotic the kernel $\tilde{U}(z_1, z_2)$ becomes a 1D projector, and its Fredholm determinant is equal to

$$\det(I - \tilde{U}) \to 1 - \oint \mathrm{d}z\, \tilde{U}(z, z). \tag{5.7}$$

Using the commutation relations between dual fields, we can evaluate $(0|\tilde{U}(z, z)|0)$ as follows

$$(0|\tilde{U}(z,z)|0) = \frac{\mathrm{e}^{-mr}}{3\pi^2 \mathrm{i}\lambda z}\sqrt{\frac{2\pi}{mr}}Y_1(z)Y_1(z^{-1}) + \cdots \tag{5.8}$$

where

$$Y_1(z) = \sum_{l=1}^{3}\omega^l(\omega^{-l}z + 1) = 3z. \tag{5.9}$$

The above result leads to the asymptotic form of the vacuum mean value of the Fredholm determinant

$$(0|\det(I - \tilde{U})|0) = 1 - \frac{\sqrt{3}}{2v^2(0)}\frac{\mathrm{e}^{-mr}}{\sqrt{2\pi mr}} + \cdots \tag{5.10}$$

which agrees with the large-distance contribution from zero- and one-particle states [4].

6. Summary

We considered the scaling Lee–Yang model and obtained the determinant representation for the correlation function of the trace of the energy-momentum tensor. We think that this representation will be useful for the study of hidden symmetries of the model. For a clear understanding of how to modify the determinant representation in order to include temperature dependence please refer to [10]. We think that the determinant representation will be useful for the evaluation of long-time asymptotic of temperature-dependent correlation functions. We believe that determinant representation is the universal language for the description of quantum correlation functions of integrable (exactly solvable) models of quantum field theory.

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